EJERCICIO ESPACIOS VECTORIALES

Determine se el conjunto V es cerrado bajo las operaciones $\oplus$ y$\odot$.

V es el conjunto de todos los pares ordenados de numeros reales (x, y), donde $x > 0$ y $y > 0$;

$$(x, y) \oplus (x\acute{}, y\acute{}) = (x + x\acute{}, y + y\acute{})$$

y

$$c\odot(x, y) = (cx, cy)$$

a) $\boldsymbol{u} \oplus \boldsymbol{v} = \boldsymbol{v} \oplus \boldsymbol{u}$

   $(x, y) \oplus (x\acute{}, y\acute{}) = (x + x\acute{}, y + y\acute{}) = (x\acute{} + x, y\acute{} + y) = (x\acute{}, y\acute{}) \oplus (x, y)$

b) $\boldsymbol{u} \oplus (\boldsymbol{v} \oplus \boldsymbol{w}) = (\boldsymbol{u} \oplus \boldsymbol{v}) \oplus \boldsymbol{w}$

   $\boldsymbol{w} = (x\acute{}\acute{}, y\acute{}\acute{})$

   $(x, y) \oplus (x\acute{} + x\acute{}\acute{}, y\acute{} + y\acute{}\acute{}) = (x + x\acute{} + x\acute{}\acute{}, y + y\acute{} + y\acute{}\acute{})$

   $(x + x\acute{}, y + y\acute{}) + (x\acute{}\acute{}, y\acute{}\acute{}) = (x + x\acute{} + x\acute{}\acute{}, y + y\acute{} + y\acute{}\acute{})$

c) $\boldsymbol{u} \oplus \boldsymbol{0} = \boldsymbol{u}$

   $(x, y) + (0, 0) = (x, y) = \boldsymbol{u}$

d) $\boldsymbol{u} \oplus - \boldsymbol{u} = 0$

   $(x, y) \oplus (-x, -y) = (0, 0) = \boldsymbol{0}$

e) $c\odot(\boldsymbol{u} \oplus \boldsymbol{v}) = c\odot\boldsymbol{u} \oplus c\odot\boldsymbol{v}$

   $c\odot[(x, y) \oplus (x\acute{}, y\acute{})] = [c(x + x\acute{}), c(y + y\acute{})]$

   $c\odot(x, y) \oplus c\odot(x\acute{}, y\acute{}) = [c(x + x\acute{}), c(y + y\acute{})]$

f) $(c + d)\odot\boldsymbol{u} = c\odot\boldsymbol{u} \oplus d\odot\boldsymbol{u}$

   $(c + d)\odot(x, y) = [(c + d)x, y]$

   $c\odot(x, y) \oplus d\odot(x, y) = (cx, y) \oplus (dx, y) = [(c + d)x, 2y]$

Por lo tanto es cerrado bajo $\oplus$ y no cerrado bajo $\oplus$.